CHAPTER 3
Linear Actuators (hydraulic cylinders)

Actuators are positive displacement units which provide linear or limited angular movement. In contrast to hydraulic motors which provide continuous movement, actuators provide reciprocating movement: linear or angular (limited rotation angle). Only linear actuators are discussed in this chapter.

The principle of operation of actuators is identical to the principle of operation of hydraulic motors. In linear actuators the element which transits the energy is a piston rod which moves with a velocity dependent on the rate of flow into the cylinder Q and piston area A. Hydraulic pressure acting on a moving element of the actuator, which seals the zone of high pressure from the zone of low pressure, results in a force and the fluid movement is translated into the velocity of the element.

In a linear actuator, as in any other positive displacement unit, there is a loss of energy which is characterized by volumetric η_v and η_{hm} efficiencies. Volumetric efficiency of an actuator η_{vc} is defined as the ratio of theoretical flow demand Q_{tc} to the actual flow demand Q_c. The actual flow rate required to drive an actuator is larger than the theoretical flow rate due to volumetric losses Q_{vc} caused by internal leakage across the piston head and external leakage through the piston rod gland seals. The modern design of actuators is characterized by excellent seals which practically eliminate both the internal and external leakage. Thus, it can be assumed that the volumetric efficiency $\eta_{vc} \equiv 1$ and $Q_c = Q_{tc}$.

For a single rod actuator, fig. 33, the speed of the piston, with assumption that $\eta_{vc} = 1$, is calculated using a continuity equation:

$$v_1 = \frac{Q_c}{A_1} = \frac{4Q_c}{\pi D^2} \tag{3.1}$$

$$v_2 = \frac{Q_c}{A_2} = \frac{4Q_c}{\pi(D^2 - d^2)} \tag{3.2}$$

Times elapsed during the full stroke of the piston are:

$$t_1 = \frac{h}{v_1} = \frac{hA_1}{Q_c} \tag{3.3}$$

$$t_2 = \frac{h}{v_2} = \frac{hA_2}{Q_c} \tag{3.4}$$

where the product of actuator stroke h and working area $A_{1(2)}$ represents actuator volume:

$$q_{1(2)} = A_{1(2)} h \tag{3.5}$$

indices 1 and 2 refer to the piston head and the annulus volumes of the actuator.

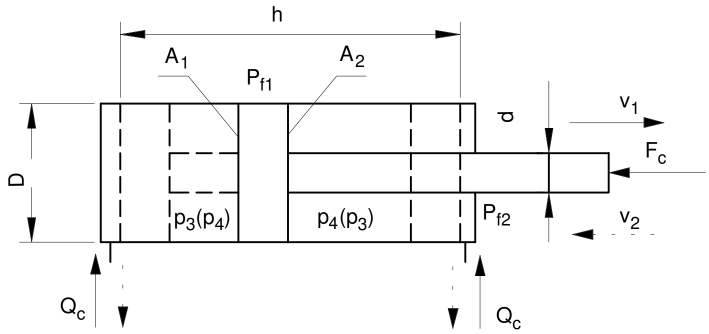


Fig. 33. A hydraulic cylinder

Modern hydraulic cylinders have very little leakage, both internal (across the piston) and external. Thus its volumetric efficiency can be assumed to be $\eta_{vc} \cong 1$. An assumption that volumetric efficiency $\eta_{vc} = 1$ implies that overall efficiency η_c is equal to hydro-mechanical efficiency η_{hmc}. The overall efficiency is calculated as the ratio of the effective output power to the input hydraulic power, thus:

$$\eta_c = \eta_{hmc} = \frac{P_h - P_{fr}}{P_h} \tag{3.6}$$

where power loss P_{fr} is due to friction of piston and gland seals and power loss due to pressure losses in actuator, its internal passages and fittings:

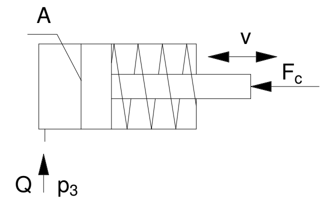
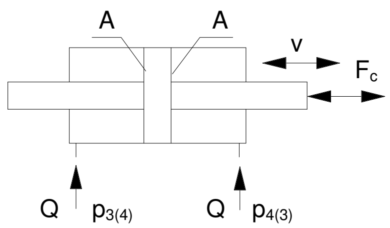
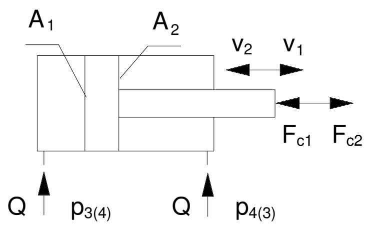
$$P_{fr} = P_{f1} + P_{f2} + P_{hl} \tag{3.7}$$

where:
P_{f1} - power loss due to piston seals friction
P_{f2} - power loss due to rod seals friction
P_{hl} - power loss due to pressure losses

The losses associated with pressure losses are significant only if the flow velocities in the passages are high, however when the actuator is correctly designed the losses are small and $P_{hl} \cong 0$. The losses due to seals friction P_{f1} and P_{f2} can be divided into losses dependent on pressure difference $\Delta p_{34} = p_3 - p_4$ across the piston and losses independent of the pressure (under normal circumstances they can be ignored).

Expressions for volumetric efficiency η_{vc} and hydro-mechanical efficiency η_{hmc} for various types of hydraulic cylinders is shown in table 7.

Table 7. Volumetric and hydro-mechanical efficiencies of actuators

Actuator type	**Efficiency** η_{vc}	**Efficiency** η_{hmc}
	$\frac{vA}{Q}$	$\frac{F_c}{Ap_2}$
	$\frac{vA}{Q}$	$\frac{F_c}{A\Delta p_{34}}$
	$\frac{v_1A_1}{Q}$ $\frac{v_2A_2}{Q}$	$\frac{F_{c1}}{A_1p_3 - A_2p_4}$ $\frac{F_{c2}}{A_2p_3 - A_1p_4}$

Output power of an actuator in fig. 33 is the product of cylinder force F_c and piston speed v. Input power is the difference between hydraulic power delivered to the actuator and hydraulic power on the return side of the actuator. Thus, actuator overall efficiency is calculated from the expression:

$$\eta_c = \eta_{hmc} = \frac{F_c v_{1(2)}}{Q_3 p_3 - Q_4 p_4} \tag{3.8}$$

where:

Q_3, Q_4 - volumetric flow rates - delivery and return ports
p_3, p_4 - pressures in delivery and return sides of the actuator

Output force F_c is equal to:

$$F_c = p_3 A_{1(2)} - p_4 A_{2(1)} - F_f \tag{3.9}$$

where F_f - total seal friction force (piston and piston rod). Thus taking into consideration that:

$$v_{1(2)} = \frac{Q_3}{A_{1(2)}} \tag{3.10}$$

The efficiency of the actuator can be defined as:

$$\eta_c = 1 - \frac{F_f}{p_3 A_{1(2)} - p_4 A_{2(1)}} \tag{3.11}$$

Examples of Calculations - Control of Actuators

Problem 3.1 Cylinder flow demand

Calculate delivery flow Q_c required to displace piston by distance $h = 300\,\text{mm}$ during the time $t = 6\,\text{s}$. Piston area $A = 50\,\text{cm}^2$ and cylinder volumetric efficiency $\eta_{vc} = 0.99$.

Answer: Piston velocity:

$$v = \frac{h}{t}$$
$$v = \frac{0.3}{6} = 0.05\ \text{ms}^{-1}$$

Required theoretical flow rate Q_{tc}:

$$Q_{tc} = Av$$
$$Q_{tc} = 5 \times 10^{-3} \times 0.05 = 2.5 \times 10^{-4}\ \text{m}^3\text{s}^{-1}$$

and actual flow rate Q_c:

$$Q_c = \frac{Q_{tc}}{\eta_{vc}}$$
$$Q_c = \frac{2.5 \times 10^{-4}}{0.99} = 2.53 \times 10^{-3}\ \text{m}^3\text{s}^{-1} \qquad \text{answer!}$$

Problem 3.2 Cylinder force and efficiency

A cylinder has the following piston areas: $A_1 = 25\,\text{cm}^2$ and $A_2 = 15\,\text{cm}^2$ (rod side). Pressure measurements showed that delivery pressure $p_3 = 10$ MPa and return pressure $p_4 = 1.5$ MPa. Calculate cylinder force during working and return strokes if hydro-mechanical efficiency of cylinder $\eta_{hmc} = 0.90$.

Answer: Theoretical force during a working stroke of the piston is:

$$F_{t1} = p_3 A_1 - p_4 A_2$$
$$F_{t1} = 10 \times 10^6 \times 25 \times 10^{-4} - 1.5 \times 10^6 \times 15 \times 10^{-4} = 22.75\ \text{kN}$$

Theoretical force during the return stroke of the piston is:

$$F_{t2} = p_3 A_2 - p_4 A_1$$
$$F_{t2} = 10 \times 10^6 \times 15 \times 10^{-4} - 1.5 \times 10^6 \times 25 \times 10^{-4} = 11.25\ \text{kN}$$

Actual forces F_{c1} and F_{c2}, at $\eta_{hmc} = 0.9$, are:

$$
\begin{aligned}
F_{c1} &= F_{t1} \times \eta_{hmc} \\
F_{c1} &= 22.75 \times 0.9 = 20.5 \text{ kN} && \text{answer!} \\
F_{c2} &= F_{t2} \times \eta_{hmc} \\
F_{c2} &= 11.25 \times 0.9 = 10.1 \text{ kN} && \text{answer!}
\end{aligned}
$$

Problem 3.3 Cylinder flow demand

A double acting, single rod cylinder, fig. 34, moves with speed $v = 5\,\text{cm}\,\text{s}^{-1}$. Calculate required flow deliveries Q_1 and Q_2 if piston area $A_1 = 20\,\text{cm}^2$, area of the annulus $A_2 = 10\,\text{cm}^2$ and volumetric efficiency $\eta_{vc} = 1$.

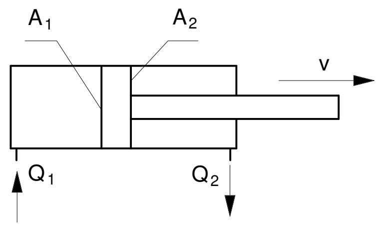


Fig. 34. Hydraulic cylinder (Problem 3.3)

Answer:

$$
\begin{aligned}
Q_1 &= A_1 v \\
Q_1 &= 20 \times 5 = 100 \text{ cm}^3\text{s}^{-1} && \text{answer!} \\
Q_2 &= A_2 v \\
Q_2 &= 10 \times 5 = 50 \text{ cm}^3\text{s}^{-1} && \text{answer!}
\end{aligned}
$$

Problem 3.4 Pressure variation in a cylinder

Calculate pressure variation in a single acting, spring return cylinder during lifting of mass $M = 71.4\,\text{kg}$, fig. 35. The spring has a linear force characteristic, the spring force varies from $F = 160$ to 300 N over the length of piston stroke. Piston area $A = 0.002\text{ m}^2$. Assume that the hydro-mechanical efficiency $\eta_{hmm} = 1$.

Answer: Force due to gravity:

$$F_g = Mg = 71.4 \times 9.81 = 700 \text{ N}$$

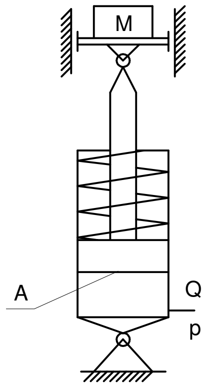


Fig. 35. Single acting, spring return cylinder (Problem 3.4)

where g is acceleration of gravity constant. Total force at the beginning of stroke:

$$F_1 = F_g + F$$
$$F_1 = 700 + 160 = 860 \text{ N}$$

and the corresponding pressure:

$$p_1 = \frac{F_1}{A}$$
$$p_1 = \frac{860}{20 \times 10^{-4}} = 4.3 \times 10^5 \text{ Pa} \qquad \text{answer!}$$

Total force at the end of stroke:

$$F_2 = 700 + 300 = 1000 \text{ N}$$

and the pressure:

$$p_2 = \frac{F_2}{A}$$
$$p_2 = \frac{1000}{20 \times 10^{-4}} = 5 \times 10^5 \text{ Pa} \qquad \text{answer!}$$

Thus the cylinder pressure varies between 430 kPa and 500 kPa.

Problem 3.5 Cylinder cycle time

For a cylinder from Problem 3.3 calculate the total cycle time if flow delivery $Q = 80\,\text{cm}^3$ and cylinder stroke $h = 32\,\text{cm}$.

Answer: The speed of piston during working stroke is:

$$v_1 = \frac{Q}{A_1}$$
$$v_1 = \frac{80}{20} = 4 \text{ cms}^{-1}$$

and elapsed time t_1:

$$t_1 = \frac{h}{v_1}$$
$$t_1 = \frac{32}{4} = 8 \text{ s}$$

During return stroke the speed v_2:

$$v_2 = \frac{Q}{A_1}$$
$$v_2 = \frac{80}{10} = 8 \text{ cms}^{-1}$$

thus elapsed time t_2:

$$t_2 = \frac{h}{v_2}$$
$$t_2 = \frac{32}{8} = 4 \text{ s}$$

thus the total cycle time:

$$t = t_1 + t_2$$
$$t = 8 + 4 = 12 \text{ s}$$

answer!

Problem 3.6 Pressure in hydraulic circuits

Calculate delivery pressure in a cylinder for each of the three circuit variants shown in fig. 36. Following data is available: piston area $A_1 = 80 \times 10^{-4}$ m^2, annulus area $A_2 = 20 \times 10^{-4}$ m^2, return pressure $p_4 = 0.2$ MPa, coefficient of friction $f = 0.2$ and hydro-mechanical efficiency of the cylinder $\eta_{hmc} = 0.9$.

Answer:

Variant a. Load acting on the piston rod: $F_c = fmg$. Theoretical piston force:

$$F_{tc} = p_3 A_1 - p_4 A_2$$

and also

$$F_{tc} = \frac{F_c}{\eta_{hmc}}$$

thus:

$$F_{tc} = \frac{fmg}{\eta_{hmc}}$$

$$F_{tc} = \frac{0.2 \times 10^4 \times 9.81}{0.9} = 21.8 \times 10^3 \text{ N}$$

Using the equation for theoretical piston force, delivery pressure p_3:

$$p_3 = \frac{F_{tc} + p_4 A_2}{A_1}$$

and

$$p_3 = \frac{21.8 \times 10^3 + 2.0 \times 10^5 \times 20 \times 10^{-4}}{80 \times 10^{-4}} = 2.78 \text{ MPa} \qquad \text{answer!}$$

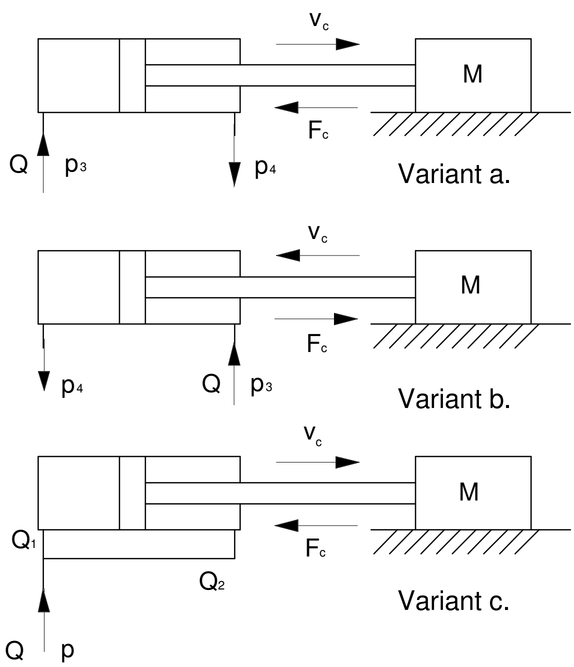


Fig. 36. Cylinder circuits (Problem 3.6)

Variant b. The theoretical force on the rod:

$$F_{tc} = p_3 A_2 - p_4 A_1$$

thus:

$$p_3 = \frac{F_{tc} + p_4 A_1}{A_2}$$

$$p_3 = \frac{21.8 \times 10^3 + 2 \times 10^5 \times 80 \times 10^{-4}}{20 \times 10^{-4}} = 11.7 \text{ MPa} \qquad \text{answer!}$$

Variant c. The theoretical force:

$$F_{tc} = p(A_1 - A_2)$$

so

$$p = \frac{F_{tc}}{A_1 - A_2}$$

$$p = \frac{21.8 \times 10^3}{80 \times 10^{-4} - 20 \times 10^{-4}} = 3.63 \text{ MPa} \qquad \text{answer!}$$

Problem 3.7 Speed of a double acting cylinder

Calculate the speed of the piston for a double acting, single rod cylinder when delivery flow $Q = const.$, and volumetric losses are negligibly small. Consider three circuit variants as in Problem 3.6. Derive an equation for rod diameter to achieve the same piston speed in each direction of motion.

Answer:

Variants a. and ***b.*** From the continuity equation piston speed:

$$v = \frac{Q}{A}$$

where A - active piston area.

For movement of the piston to the right (***variant a***):

$$v_1 = \frac{Q_1}{A_1}$$

and for the movement to the left (***variant b***):

$$v_2 = \frac{Q_2}{A_2}$$

Piston and annulus area are:

$$A_1 = \frac{\pi D^2}{4}$$
$$A_2 = \frac{\pi (D^2 - d^2)}{4}$$

where:
D - piston diameter
d - piston rod diameter
then expressions for velocities are:

$$v_1 = \frac{4Q_1}{\pi D^2} \qquad \text{answer!}$$
$$v_2 = \frac{4Q_2}{\pi (D^2 - d^2)} \qquad \text{answer!}$$

If $Q = Q_1 = Q_2$ then:

$$\frac{v_1}{v_2} = \frac{D^2 - d^2}{D^2} = 1 - \frac{d^2}{D^2}$$

Variant a. When volumes on both sides of the piston are connected together the piston will move to the right due to the difference of areas $A_1 - A_2 > 0$. Delivery flow Q is increased by the flow Q_a from the annulus volume of the cylinder:

$$Q_a = v_3 A_2$$

thus

$$v_3 = \frac{Q + Q_a}{A_1} = \frac{Q + v_3 A_2}{A_1}$$

and

$$v_3 = \frac{Q}{A_1 - A_2} = \frac{4Q}{\pi D^2 - \pi (D^2 - d^2)} = \frac{4Q}{\pi d^2} \qquad \text{answer!}$$

To obtain the same speed in both directions of piston movement, for the piston movement to the left the cylinder should be connected according to ***variant b.***, and for the movement to the right according to ***variant c.*** Using expressions for v_2 and v_3 the equality condition $v_2 = v_3$ can be written as:

$$\frac{Q}{A_2} = \frac{Q}{A_1 - A_2}$$

so

$$A_2 = A_1 - A_2$$

and

$$A_1 = 2A_2$$

Substituting expressions for A_1 and A_2

$$\frac{\pi D^2}{4} = \frac{2\pi(D^2 - d^2)}{4}$$

we obtain condition for equal speed in both directions of movement, ***variant b.***and ***c.***:

$$d = \frac{\sqrt{2}}{2} D \qquad \text{answer!}$$

Problem 3.8 Cylinder size

Calculate piston diameter D, rod diameter d and actuator pressure p_3 for a double acting, single rod cylinder using the following data :

piston force	$F_c = 80 \times 10^3$ N
supply pressure (piston head)	$p_3 = 16$ MPa
return pressure (annulus)	$p_4 = 0.3$ MPa
ratio of areas	$\gamma = A_1/A_2 = 1.25$
hydro-mechanical efficiency of the cylinder	$\eta_{hmc} = 0.95$

Answer: Using equation for hydro-mechanical efficiency of a cylinder we obtain:

$$A_1 = \frac{F_c}{\left(p_3 - \dfrac{p_4}{\gamma}\right)\eta_{hmc}} \qquad \text{(a)}$$

$$A_1 = \frac{80 \times 10^3}{\left(16 \times 10^6 - \dfrac{0.3 \times 10^6}{1.25}\right) 0.95} = 5.34 \times 10^{-3}\,\text{m}^2$$

and then diameter of the piston is equal to:

$$D = \sqrt{\frac{4A_1}{\pi}}$$

$$D = \sqrt{\frac{4 \times 5.34 \times 10^{-3}}{\pi}} = 82.5\,\text{mm} \qquad \text{answer!}$$

Using a preferred nominal diameter $D = 80\,\text{mm}$ piston and annulus areas are:

$$A_1 = \frac{\pi\left(80 \times 10^{-3}\right)^2}{4} = 5.03 \times 10^{-3}\,\text{m}^2$$

$$A_2 = \frac{A_1}{\gamma} = \frac{5.03 \times 10^{-3}}{1.25} = 4.02 \times 10^{-3}\,\mathrm{m}^2$$

thus diameter of piston rod is:

$$d = \sqrt{D^2 - \frac{4A_2}{\pi}}$$

$$d = \sqrt{(80 \times 10^{-3})^2 - \frac{4 \times 4.02 \times 10^{-3}}{\pi}} = 35.8\,\mathrm{mm} \qquad \text{answer!}$$

and we choose the preferred value $d = 36\,\mathrm{mm}$. Using values for D and d we can calculate pressure necessary to obtain force $F_c = 80 \times 10^3$ N. As the new value of A_2 is:

$$A_2 = \frac{\pi}{4} \times \left[\left(80 \times 10^{-3}\right)^2 - \left(36 \times 10^{-3}\right)^2\right] = 4.01 \times 10^{-3}\,\mathrm{m}^2$$

then, again using equation (a) and rearranging, required delivery pressure p_3 is:

$$p_3 = \frac{F_c + p_4 A_2 \eta_c}{A_1 \eta_c}$$

$$p_3 = \frac{80 \times 10^3 + 0.3 \times 10^6 \times 4.01 \times 10^{-3} \times 0.95}{5.03 \times 10^{-3} \times 0.95} = 16.9\,\mathrm{MPa} \qquad \text{answer!}$$

The calculated pressure is 12% higher than specified above, thus if for a given system this pressure is not acceptable we would have to choose a large piston diameter and repeat the calculations.

Problem 3.9 Speed of a telescopic cylinder

A two-stage telescopic cylinder, shown in fig. 37, is loaded by force F. Plot the speed of piston **3** as a function of its displacement x. Assume that delivery flow $Q = const.$ and that volumetric and hydraulic losses are negligibly small.

Answer: The rigidly mounted main cylinder **1** and cylinder **2** form the first stage of the telescopic cylinder. Cylinder **2** and piston **3** form the second stage of the cylinder. Pressure $p_{3(1)}$ at which piston **3** will start moving may be calculated from the equation for a balance of forces:

$$p_{3(1)}A_3 - p_4A_4 - F_{fD2} - F_{fd2} - F = 0$$

Areas A_3 and A_4 are:

$$A_3 = \frac{\pi D_2^2}{4}$$

$$A_4 \quad = \quad \frac{(D_2^2 - d_2^2)}{4}$$

where:

F_{fD2}, F_{fd2}	-	friction forces on diameters D_2 and d_2
F	-	external force

and from the above:

$$p_{3(1)} = \frac{F + p_4 A_4 + F_{fD2} + F_{fd2}}{A_3}$$

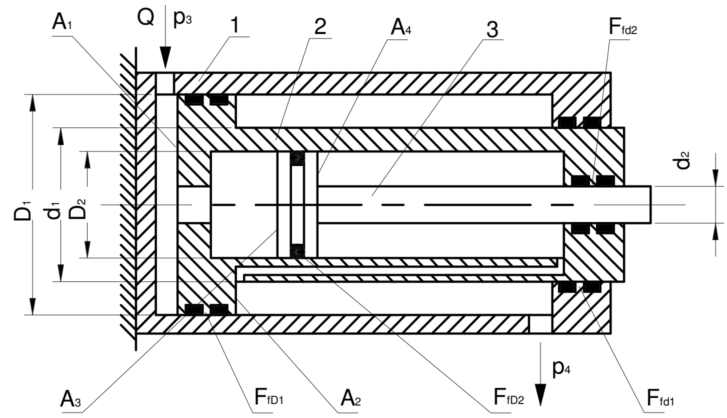


Fig. 37. Telescopic cylinder (Problem 3.9)

The speed of piston **3** relative to cylinder **2** is calculated from relation:

$$v_1 = \frac{Q}{A_3}$$

Pressure $p_{3(2)}$ at which cylinder **2** and piston **3** will start moving is again calculated from the balance of forces equation:

$$p_{3(2)} A_1 - p_4 A_2 - F_{fD1} - F_{fd1} - F = 0$$

where $F_{fD1}, F_{fd1}$are friction forces on diameters D_1 and d_1 and areas A_1 and A_2 are:

$$A_1 \quad = \quad \frac{\pi D_1^2}{4}$$
$$A_2 \quad = \quad \frac{\pi (D_1^2 - d_1^2)}{4}$$

thus:

$$p_{3(2)} = \frac{F + p_4 A_2 + F_{fD1} + F_{fd1}}{A_1}$$

The speed of cylinder **2** is equal to:

$$v_2 = \frac{Q}{A_1}$$

The order in which the components of the cylinder will start moving depends on the magnitude of pressure required to move these components. By design $A_1 > A_2$, $A_3 > A_4$ and the value of force F is significantly higher than value of friction forces and back pressure, thus:

$$\begin{aligned} F &\gg p_4 A_4 + F_{fD2} - F_{fd2} \\ F &\gg p_4 A_2 + F_{fD1} + F_{fd1} \end{aligned}$$

and in general:

$$p_{3(2)} < p_{3(1)}$$

so, if $p_{3(2)} < p_{3(1)}$ then cylinder **2** will start moving first with speed v_2 and after it reaches its end position piston **3** will start moving with speed v_1. The plot of speed of piston **3** in function of its position is shown in fig. 38, where:

h_1 - stroke of the second stage - cylinder **2** relative to cylinder **1**
h_2 - stroke of piston **3** relative to cylinder **2**
H - total stroke of telescopic cylinder

Under condition $F = 0$ the values of pressures $p_{3(1)}$ and $p_{3(2)}$ are affected only by friction forces and the value of back pressure p_4.

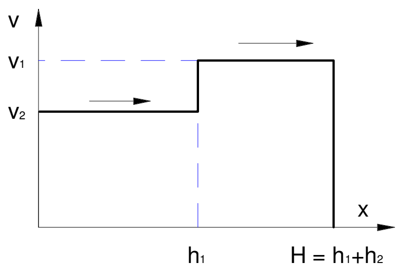


Fig. 38. Speed of telescopic cylinder vs. its stroke (Problem 3.9)

Problem 3.10 Cylinder cushion

To prevent hard bottoming of the piston in a cylinder, a piston damping arrangement (a hydraulic cushion), shown in fig. 39, is provided. Calculate the required length of spigot **1** to prevent the piston hitting the bottom of the cylinder. Steady-state velocity of the piston before piston enters cushion is $v_0 = 0.5\ \mathrm{m^3 s^{-1}}$. Use the following data:

mass	$M = 10 \times 10^3\,\mathrm{kg}$
diameters of the piston	$D = 120\,\mathrm{mm}$,
diameter of the spigot	$D_c = 40\,\mathrm{mm}$
radial clearance	$e = 0.4\,\mathrm{mm}$
dynamic viscosity	$\mu = 0.02\ \mathrm{Nsm^{-2}}$

Assume that back pressure p_4 and the losses in the cylinder are negligibly small, and that pressure p_3 is constant. Use the following equation for flow through the clearance

$$Q_h = \frac{\pi D_c e^3}{12\mu x}(p_h - p_4)$$

where: p_h - pressure in damping volume

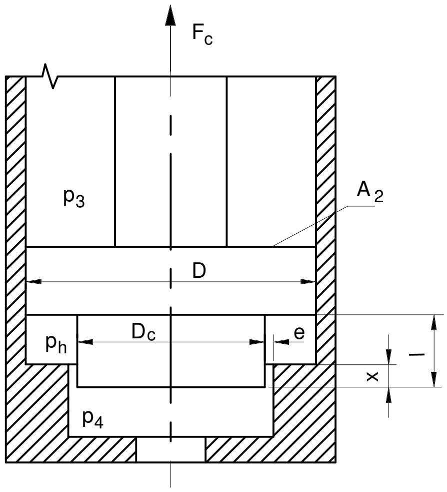


Fig. 39. Cylinder damping (Problem 3.10)

Answer: The equation for the piston deceleration is:

$$p_3 A_2 - p_h A_h - F_c = M\frac{d^2x}{dt^2} \tag{a}$$

and flow through the clearance is described by relation:

$$Q_h = \frac{dx}{dt}A_h = \frac{\pi D_c e^3}{12\mu x}(p_h - p_4) \tag{b}$$

where:

A_h - annulus area, $A_h = \dfrac{\pi\left(D^2 - D_c^2\right)}{4}$

In accordance with the assumption that pressure p_3 is constant (as set by a relief valve) the following relation is valid:

$$p_3 A_2 - F_c = 0 \tag{c}$$

and thus from eq. (a) we obtain value of p_h:

$$p_h = -\frac{M}{A_h}\frac{d^2x}{dt^2} \tag{d}$$

We substitute for p_h in (b) and as $p_4 \approx 0$ then the equation for piston movement during braking is:

$$\frac{\pi M D_c e^3}{12\mu A_h^2}\frac{d^2x}{dt^2} + x\frac{dx}{dt} = 0 \tag{e}$$

We define constant k, which after substitution for A_h is equal to:

$$k = \frac{\pi M D_c e^3}{12\mu A_h^2} = \frac{4 M D_c e^3}{3\mu\pi\left(D^2 - D_c^2\right)^2}$$

$$k = \frac{4 \times 10 \times 10^3 \times 0.04 \times \left(0.4 \times 10^{-3}\right)^3}{3 \times 0.02 \times \pi \times \left(0.12^2 - 0.04^2\right)^2} = 3.32 \times 10^{-3} \text{ ms}$$

and write equation (e) as:

$$k\frac{d^2x}{dt^2} + x\frac{dx}{dt} = 0 \tag{f}$$

considering that:

$$\frac{dx}{dt} = v$$

and also:

$$\frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

thus, after separating the variables, we obtain equation:

$$k dv = -x dx$$

and integrating both sides we obtain:

$$\int_{v_0}^{v} k dv = -\int_{x_0}^{x} x dx$$

$$k(v - v_0) = -\frac{1}{2}x^2$$

thus finally the equation for speed of piston is:

$$v = v_0 - \frac{x^2}{2k} \qquad \text{(g)}$$

Equation (g) describes the speed of the piston during braking in function of piston displacement x. At the end of the braking phase piston speed $v = 0$, thus we may calculate braking distance x_h:

$$x_h = \sqrt{2kv_0} = \sqrt{2 \times 3.32 \times 10^{-3} \times 0.5} = 5.8 \times 10^{-2}\,\mathrm{m}$$

To avoid the piston hitting the end cover, the spigot length must be longer than x_h, thus:

$$l \geqslant 0.058\,\mathrm{m} \qquad \text{answer!}$$

We may obtain time functions of displacement $x(t)$, velocity $v(t)$, and also pressure $p_h(t)$ by solving eq. (f) and also using eq. (d):

$$x(t) = x_h \tanh(\alpha t)$$

$$v(t) = v_0(1 - \tanh^2(\alpha t))$$

$$p_h(t) = \frac{M v_0 \alpha}{A_h}(\tanh(\alpha t) - \tanh^3(\alpha t))$$

where $\alpha = \sqrt{\frac{v_0}{2k}}$. Plots of $x(t)$, $v(t)$ and $p_h(t)$ using data for the damping system are shown in figs 40, 41 and 42.

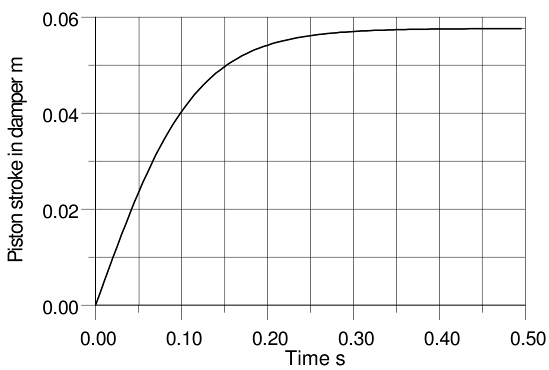


Fig. 40. Plot of piston stroke in damper

Using the damper the piston slows down in approximately 0.4 s, pressure in the damping chamber reaches maximum of approximately 3.3 MPa.

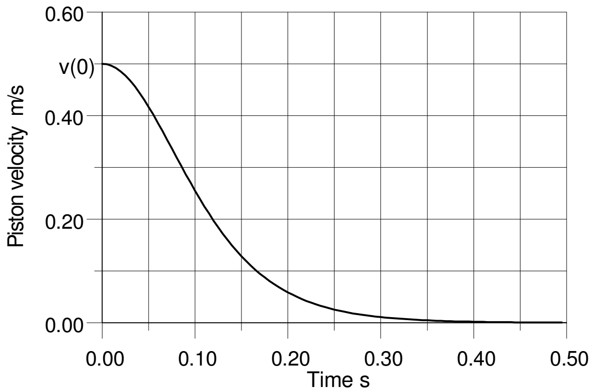


Fig. 41. Plot of piston velocity

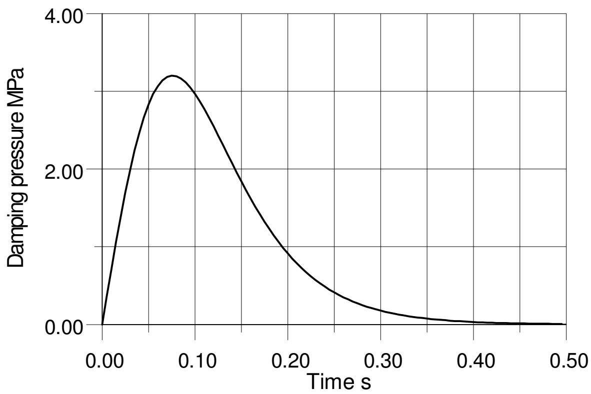


Fig. 42. Pressure in damping volume